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United States  
Department  
of Agriculture

Forest Service

Intermountain  
Research Station

Research Note  
INT-395

February 1991



# Statistical Procedures for Disaggregation Applicable to Modeling Climatic Effects on Forest Growth<sup>1</sup>

Albert R. Stage<sup>2</sup>

## ABSTRACT

*Many processes that are observable for an entire population can be difficult to observe for the individuals in the population. In a conceptually similar problem, a production process may be influenced by factors that vary in a temporal sequence during production, but the yield is observable only at the completion of the process. In this note, statistical procedures for disaggregation, originally developed by R. A. Fisher, are reintroduced with suggestions for how they might be applied to problems of modeling effects of climate and stand structure on forest growth.*

**KEYWORDS:** tree growth response, stand structure

How resources are allocated among individuals in populations is an important problem in modeling ecosystems. Biophysical theory, although helpful, is often incomplete, leaving critical parameters to be determined. Available radiation, nutrients, and moisture can be measured with relative ease on an area basis. How much is available to and used by individual trees occupying that area is either unobservable or very expensive to estimate. A conceptually similar problem arises when total production can be measured only at the end of a period spanning several stages of development, each of which

may be influenced by varying conditions of environment. The purpose of this presentation is to reintroduce some statistical procedures that are applicable to the problem of disaggregation of effects.

Applications of statistical procedures for disaggregating effects include estimating the effects of daily precipitation on seasonal yield (Fisher 1925; Schumacher and Day 1939; Schumacher and Meyer 1937), allocating milling costs to log size when only daily production totals and costs can be observed (Schumacher and Jones 1940), allocating growing space to individual trees when only stand totals can be determined (Chisman and Schumacher 1940; Curtis 1971; Lexen 1939), or estimating contributions of different stand types to an inventory of dead timber when measurements are only available for compartment totals (Stage unpublished). All of these disparate examples share a common element. In each, the totals are available, but the individual contributions are either unobservable or prohibitively expensive to measure. Furthermore, in each case, an additive model is intrinsically reasonable.

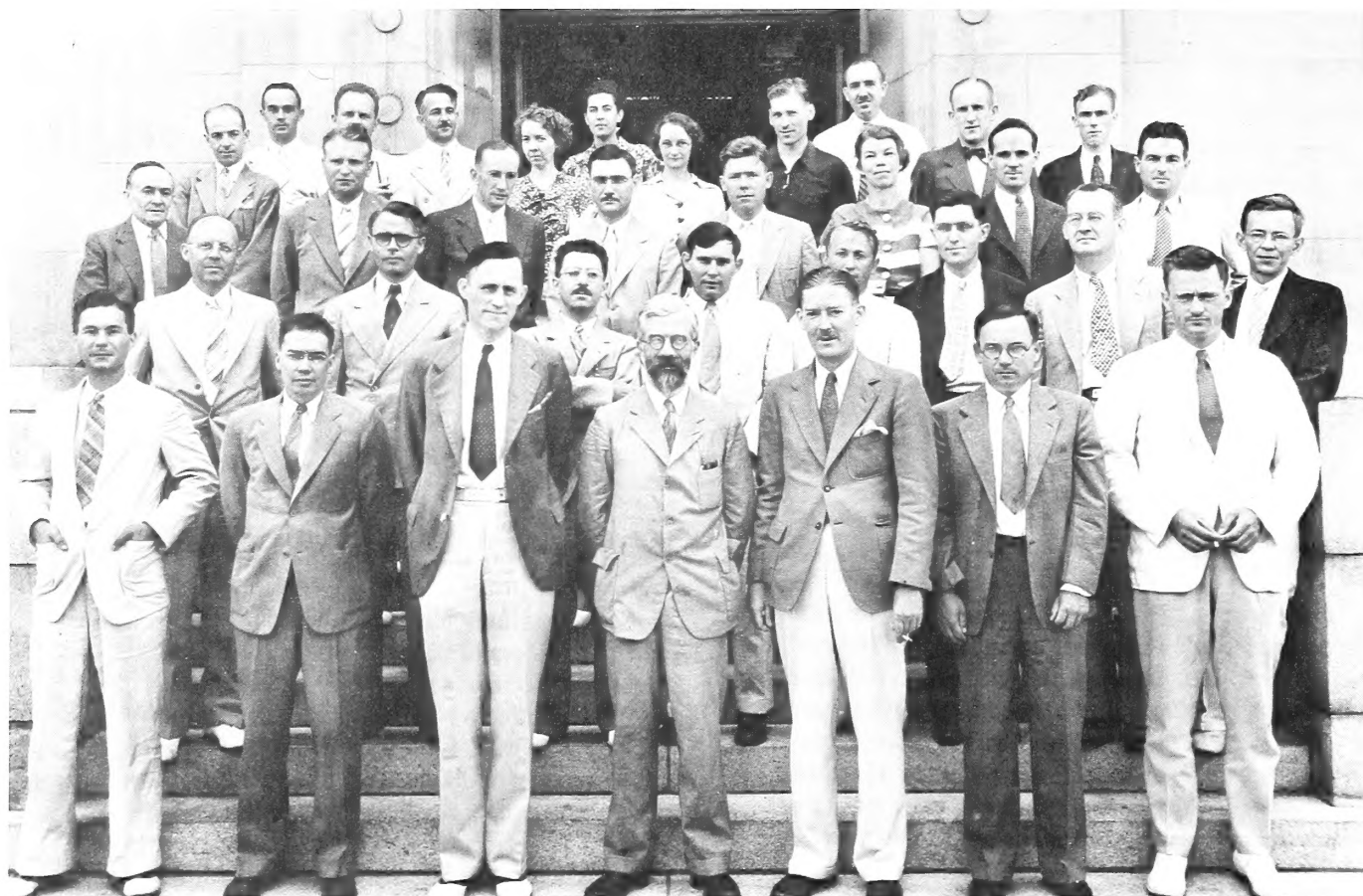
## HISTORICAL ROOTS

The seminal work addressing the problem of disaggregation is a paper by R. A. Fisher in 1925 titled "The Influence of Rainfall on Yield of Wheat at Rothamsted." From that origin, the trail leads to a statistical training seminar in 1936 for Forest Service Research staff at which Fisher was a featured lecturer. A group photo taken at that seminar (fig. 1) includes at least three of the authors of applications of Fisher's method—F. X. Schumacher standing to Fisher's left, with Bessie Day, and Bert Lexen immediately above them.

The procedure, although by no means new, seems still to have application for constructing models with which to evaluate effects of a globally changing environment.

<sup>1</sup>Based on presentation at workshop: Forest/Climate Interactions: Scale Transition from Individual Leaves to the Global. Sponsored by National Center for Atmospheric Research and USDA Forest Service, September 25-27, 1990, Boulder, CO.

<sup>2</sup>Principal Mensurationist and Project Leader, Quantitative Analysis of Forest-Resource Responses for Management, Planning, and Control, located at Moscow, ID.



More than a score of forestry experts from experiment stations of the National Forest service of the United States are here attending a meeting at the federal building. They came here to confer with Dr. R. A. Fisher, of London, England, Fellow of the Royal Society and professor of the London University. The meeting will adjourn this week. Those attending are shown above. First row, left to right: Dr. G. H. Hepting, Asheville; Dr. I. T. Haig, Washington, D. C.; C. L. Forsling, director of the Appalachian station here; Dr. Fisher; F. X. Schumacher, Washington, D. C.; A. A. Hasel, Berkeley, Calif.; and T. S. Coile, Duke University, Durham. Second row, left to right: F. P. Keen, Washington, D. C.; V. L. Harper, New Orleans, La.; G. R. Gevorkiantz, St. Paul, Minn.; R. A. Chapman, New Orleans, La.; B. R. Lexen, Tucson, Arizona; J. H. Buell, Asheville; Dr. W. H. Meyer, Seattle, Washington; and Dr. A. L. MacKinney, Asheville. Third row, left to right: E. H. Frothingham, Asheville; R. A. Price, Ogden, Utah; K. P. Davis, Missoula, Montana; I. H. Sims, Washington, D. C.; J. G. Osborne, New Orleans, La.; Miss Bessie Day, Berkeley, Calif.; R. H. Blythe, St. Paul, Minn.; and C. V. Roberts, Asheville. Back row, left to right: L. E. Chaiken, Asheville; B. H. Wilford, Asheville; L. I. Barrett, Asheville; Dr. C. R. Hursh, Asheville; Miss Lillian Taylor, Washington, D. C.; Miss Nancy DeLachmutt, Washington, D. C.; Miss Dorothy Johnson, Asheville; L. H. Reineke, New Haven, Conn.; G. L. Schnur, Philadelphia, Pa.; C. A. Connaughton, Fort Collins, Colorado; and C. A. Bickford, Asheville.

**Figure 1**—Group of Forest Service Research staff attending Statistical Seminar at which Dr. R. A. Fisher was featured lecturer. Photo courtesy of Dr. Larry Davis. Captions supplied by Dr. Roger Chapman (taken from a newspaper account).

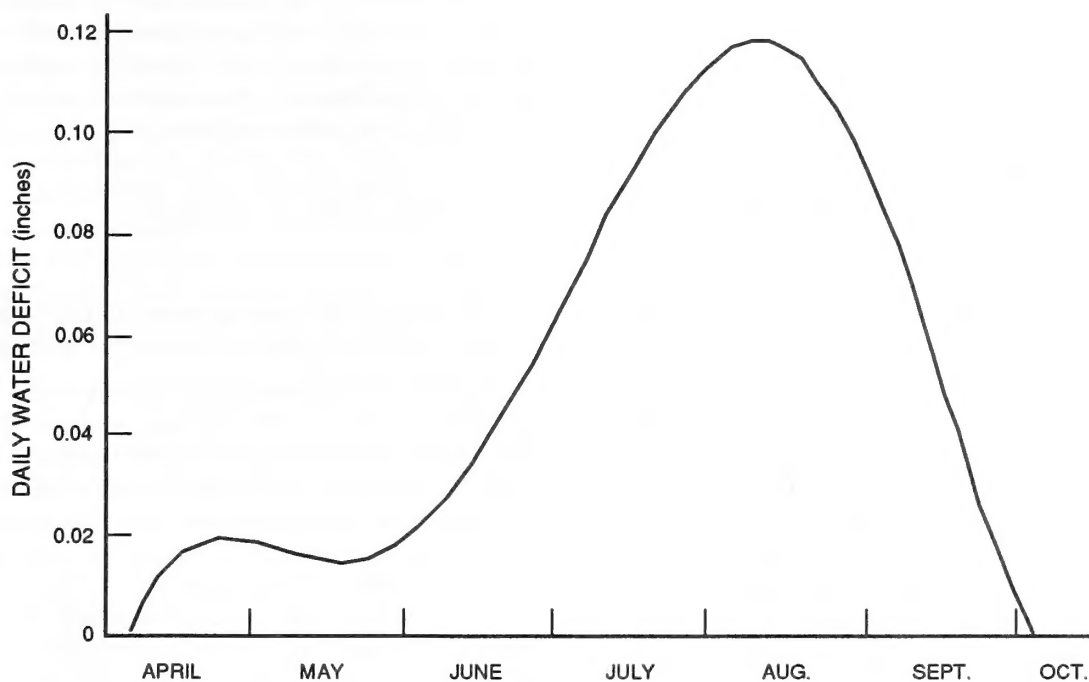
## DESCRIPTION OF PROCEDURE

The first step in the disaggregation procedure is to summarize the distribution of the entities contributing to the total. Several applications have used the first three moments of the diameter distribution (of trees in a stand, logs on a rail car, or logs processed each day in a mill). When the production units are arbitrary subdivisions of a continuous scale, Fisher (1925), Schumacher and Day (1939), Schumacher and Meyer (1937), and Zahner and Stage (1966) used coefficients of orthogonal polynomials describing the sequence of values (for example, daily rainfall during the crop year, daily moisture stress, or daily temperatures). For example, figure 2 shows a daily trend of water deficits as represented by a fifth-degree orthogonal polynomial. Seasonal variation of this attribute from year to year would be represented by variation in the coefficients of the polynomial fit to the data for each year. Application of the procedure differs between temporal and spatial problems.

## Disaggregation Into a Temporal Sequence

In Fisher's original formulation, he argued that the effect of rainfall on wheat yield could be represented by a multiplier of daily rainfall that changed slowly and continuously as the season progressed, and that furthermore, the multiplier could be represented as a low order polynomial in time. For ease of interpretation (and computation!), use of orthogonal terms permitted one to evaluate the linear effect, and then effects of each successively higher order independent of lower order effects. With the advent of accurate electronic computing, interest in orthogonal polynomials has waned, although I believe loss of the sequential interpretive advantages is to be regretted.

The second step is to estimate the coefficients in the model. Although Fisher and his followers did not describe the solution of the normal equations as a regression problem, the calculations can in fact be performed with standard regression procedures.



**Figure 2**—Daily trend of water deficits for the 1958 growing season at the Priest River Experimental Forest, represented by fifth-degree polynomial. Site stores 4 inches of available water at field capacity.

To illustrate the method of disaggregation of seasonal growth into daily contributions, consider growth as a phenomenon that starts and is completed in four natural intervals (as shoot flushing, needle elongation, bud setting, and food storage, for example). Then it would be natural to explain the growth ( $y$ ) as the sum of the contributions from each of the intervals. And in turn, the contribution in an interval is proportional to the causative factor,  $s_i$  in the  $i$ th interval.

The model is then:

$$y = \sum_{i=1}^4 a_i \cdot s_i \quad (1)$$

and the coefficients ( $a_i$ ) of proportionality would be estimated by a least-squares method.

If the intervals are small relative to the stages of development then it is reasonable to assume that these coefficients of proportionality change from interval to interval in a smooth fashion. In that case, they might be represented by a low-order polynomial in time. For example, we might assume a second-degree polynomial:

$$a_i = c_0 + c_1 i + c_2 i^2 \quad (2)$$

$i = 1, \dots, n$

where  $n$  is the number of time intervals and the  $c_i$ 's are coefficients in the polynomial describing the changing effect through time.

Substituting this estimate of the constants in the original model (1):

$$y = \sum_{i=1}^n (c_0 + c_1 i + c_2 i^2) s_i \quad (3)$$

$$y = c_0 \sum_{i=1}^n s_i + c_1 \sum_{i=1}^n i s_i + c_2 \sum_{i=1}^n i^2 s_i \quad (4)$$

For perennial species and especially trees, there is a pronounced ante-dependence in the growth response. That is, the growth of the individual in the year previous is, per se, predictive of the growth in the current year. For example, if growth is poor one year because of adverse weather during that year, then growth will tend to be poor in the following year as well, independent of growing conditions of the following year. Current growth is governed not only by environment but also by preconditioned physiological and anatomical characteristics that may have been influenced by previous growing conditions.

Thus, the model appropriate for perennial plants is somewhat more complex than that described by Fisher, Schumacher, and their colleagues. Consider the problem of evaluating current growth ( $y_i$ ) using only one weather factor, say moisture stress ( $S_i$ ), where  $i$  is the index of year of growth. Then the model including effect of previous year's growth

( $y_{i-1}$ ) and the weather factors in both years is:

$$y_i - b_1 y_{i-1} = B_2 S_i + B_3 S_{i-1} + \epsilon_i \quad (5)$$

where uppercase symbols are used to represent vector-valued variables.  $B$  and  $b$  are regression coefficients.

In time series terminology, this model would be an autoregressive, moving average (ARMA) model but for one important difference (Monserud 1986). In the usual ARMA models, the  $S$ 's on the right side would be random, unobservable shocks to the system. In our case, they are observable or separately calculated as in Zahner and Stage (1966). Because of this structure of the right-hand side, the model is, in the terminology of econometrics, a distributed-lag model.

## Disaggregation to Discrete Units

To illustrate the disaggregation to discrete units, consider the estimation of growing space requirements (Chisman and Schumacher 1940; Lexen 1939). The estimate of area occupied is based on the hypothesis that the growing space occupied by a single tree (and hence its share of the site resources) could be represented as a second degree function of tree diameter. Then the total growing space, which is just plot area, is the sum of growing space of the  $n$  individual trees, so:

$$A = b_0 \sum_{i=1}^n 1 + b_1 \sum_{i=1}^n d + b_2 \sum_{i=1}^n d^2 \quad (6)$$

where

$A$  = total land area occupied by the  $n$  trees, and  
 $d$  = diameter at breast height of each tree.

The  $b$ 's were estimated by least squares. Chisman and Schumacher (1940) further partitioned the sum by growth habit and species, and Curtis (1971) replaced the polynomial with a power function, but the same procedure applies. When applying this technique, the choice of plots should include stands of widely different structure and sizes of trees, but with full site utilization assured by including understory plants and not measuring recently disturbed sites.

## CRITIQUE AND OPPORTUNITIES FOR APPLICATION TO GLOBAL CHANGE MODELING

I believe this methodology can solve some of the disaggregation problems in Global Change modeling. There are, however, several respects in which modern insight into model formulation should offer improvement.



## Temporal Disaggregation Applications

First, for problems of disaggregating temporal effects of weather within a growing season, I would substitute growing degree-days for Julian date. For example, initiation of height increment of western white pine depends on spring temperatures measured by accumulation of degree days. Therefore, to estimate effects of moisture stress on height increment, the relation should be more accurate if moisture stress was measured at the same phenological stage of development in each year rather than at the same Julian date.

Next, the independent variables representing weather effects should be reconsidered. Zahner and Stage (1966) and Baier and Robertson (1968) were able to show that realistic modeling of soil moisture stress added information beyond the explanatory power resident in the raw weather data from which the soil moisture budget is calculated. Similarly, further transformation of soil-moisture stress into an index of total photosynthesis may improve the explanatory power of the expression even more.

The temporal disaggregation process could be applied to estimate the contribution of daily photosynthesis and moisture stress for the area to annual increments in height and diameter and to changes in crown length. For this step, data collected by Monserud and Rehfeldt at the Intermountain Research Station's Forestry Sciences Laboratory in Moscow would be ideal. They have stem analysis data describing yearly increments of height, diameter, volume, and height to crown base for three coniferous species. Their sample trees are in close proximity to the Priest River Experimental Forest weather station from which observations are available since 1911, spanning most of the life of the sampled trees.

When it comes to estimation, traditional least squares methods should be replaced by more appropriate methods of cross-spectral analysis that recognize the time-series nature of the data. In such an analysis, the autoregressive moving-average effects would also be estimated.

With this approach, it will be possible to estimate the contribution of daily photosynthesis to the seasonal volume increment of the sample trees. This information would, in turn, be very useful in resolving the problem of carbon allocation between tops and roots of trees.

## Spatial Disaggregation Applications

**Leaf Area of Individual Trees**—For a more modern application of spatial disaggregation, we might be interested in allocating leaf area to individual trees. Estimates of stand totals can be

obtained from remotely sensed data (Running and others 1986) or from radiometric measurements beneath the canopy (Pierce and Running 1988). While not as appealing as direct modeling of leaf area of single trees (Gholz and others 1979), the disaggregation procedure implicitly incorporates within-stand interactions and correlations of errors.

**Photosynthesis for Individual Trees**—Some models of the photosynthetic process treat the forest canopy as a homogeneous, three-dimensional leaf (for example, Running and Coughlan 1988). Disaggregation of area estimates of photosynthesis and respiration to estimates for individual trees would bridge a gap between models at the landscape or stand level and models used to evaluate silvicultural alternatives and to translate stand growth into economic terms—applications that usually require individual tree resolution. For this purpose, however, the procedure should be modified to take advantage of new information and statistical methods. One possibility would be to replace the polynomial of tree diameter with expressions that represent leaf area (Geron and Ruark 1988) and which might incorporate additional variables such as sapwood basal area. In addition, Wyckoff (1990) has shown that growth of individual trees depends on competition measured as a sum of growing stock in trees larger than the subject tree. An expression that represents competitive effects on growth should also be applicable to estimating competition for resources.

With the modifications suggested above, one could disaggregate estimates of total stand photosynthesis into contributions from  $n$  individual trees (per unit of land area) using the following model:

$$TP = \sum^n \hat{P}_i = b_0 n + b_1 \sum^n \widehat{LA}_i - b_2 \sum^n \widehat{LAL}_i \quad (7)$$

where

$TP$  = photosynthesis estimated for total stand,

$P_i$  = estimated allocation of daily photosynthesis to  $i$ th tree

$\widehat{LA}_i$  = leaf area of  $i$ th tree estimated from relations such as described by Gholz and others (1979)

$\widehat{LAL}_i$  = sum of leaf-area estimates over trees in stand larger than the subject tree

and the  $b_i$  are estimated by econometric "errors in variables" methods (Johnston 1960) that are more efficient when the independent variables are measured with stated errors.

If the seasonal dynamics of leaf-area and photosynthesis are not constant, one could further postulate that the  $b$ 's change with time within season as a low-order polynomial in growing degree-days. Then, the methods described above could be invoked.

One advantage of this formulation is that it obviates the need for (or at least supplements) approximations of light competition by Beer's law. Furthermore, it implicitly encompasses differential effects of location within the canopy on wind movement, temperature gradients, and competition for soil moisture as well as the attenuation of light. Analyses such as these would provide a cross-check on strictly physiological models that represent photosynthesis at the level of individual trees. Through comparison, improvements in both approaches should become apparent.

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Intermountain Research Station  
324 25th Street  
Ogden, UT 84401



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